## *Groups in Nature*

You may be out hiking on a beautiful mountain trail, strolling along the beach, or just walking along the irrigation canal that runs through your city. Have you ever had the feeling that you were part of your surroundings? I mean, not just observing them, but that you were actually a part of them and they were a part of you?

How can this be? Well, it happens on physical, emotional and spiritual levels. You breath the air, drink the water and consume locally grown produce, and they become part of your body in a very real way. You breathe out carbon dioxide and return other nutrients that the plants absorb. In this way you become part of them. Emotions flow in the air and seem to have an existence of their own. Maybe it's just the warm sun on your skin or - who knows.

Have you ever experienced a heightened sense of awareness or a special connection with all of nature around you? That's the "flow state" or being "in the zone." It happens when everything is just right and you have finally let go of all cares!

> If you have been there,
> You know about the "magic moments"
> That only a few will ever discover!

When I am backpacking in the Sierra Nevada Mountains of California I am overwhelmed with peace and joy! Many times I feel like I am not actually hiking on the trail, but floating about a foot off the ground! Maybe it's endorphins, maybe it's something else, but there are magic moments up in the mountains just waiting for me to return!

It's as if part of me lives there all the time. When I return to the mountains I am reunited with that part, and am whole.

All this beauty and inspiration! Can it relate to something as austere and technical as mathematics? Absolutely! In fact, mathematics isn't austere.

> The heart of mathematics is pattern
> And its soul is beauty!

There are gatekeepers who would like to keep you out, but I am inviting you in!

This may seem strange, but some of our experiences in nature can be understood using a mathematical construct from Abstract Algebra call the "Group."

It's a natural thing that relates to the simplicity and joy of experiencing nature. Groups don't have to be complicated or even just deal with numbers. There are "non-numeric" groups that describe actions, and even thoughts and concepts! Who would have thought a mathematical construct could, even in a small way, describe our enjoyment of nature?

How amazing!

So, let's take a hike.

You may think this journey is something like Alice going down the Rabbit Hole. Well, maybe in some ways it is, but this adventure is just one small facet of the joy to be had exploring a study called Abstract Algebra.

While out hiking in nature you may be soaking up all the wonderful sights and smells around you. You may be just enjoying the thrill of how your body reacts to exercise (especially you younger people). You probably look to the right, look to the left, and sometimes look behind you. You may want to just sit for a while and enjoy the peace and serenity all around you.

These actions, together with the operation "followed by" form a mathematical Group! Now, here's a powerful idea to consider:

> Once you classify a set of elements as a Group,
> You automatically know a bunch more things about them!
> You understand them in a new and expanded way!
> As with everything in nature,
> The whole is more than the sum of the parts!

So, let me define what a Group is, and why it's important to recognize groups in nature. Don't be turned off by a few technical terms. It's really not hard to understand. In fact it's pretty simple! And here's the important thing:

> These concepts will open up whole new levels of understanding,
> Amazement, and joy about
> The natural world that we live in!

So, hang in there for the definition, and then we'll have some fun discovering groups in nature.

Here's the definition of a Group. Ready?

## *Definition of a Group*

A Group consists of two things:

**1. A set of elements**: These elements can be almost anything - not just numbers.

**2.An operation that acts on those elements**: Just to be a little bit abstract, we'll use the asterisk (*) as the symbol for the operation.

Now, this set of elements together with the operation will constitute a Group **if** they meet the following requirements:

**1. Closure:** The group has to be closed.

**2. Identity Element:** There has to be an identity element.

**3. Inverse:** Each element has an inverse.

**4. Associativity:** The group has to be associative.

That might sound kind of technical, BUT it's really easy. No kidding!

Let's have a look at this definition. Once you understand it, you have it made! Then the fun begins, and you will be searching for groups everywhere!

So, what are these requirements all about?

## *1. Closed*

OK, this one's pretty easy. When the operation is applied to any two members of the group, the result is still in the group.

For example, let's say we're talking about the set of even numbers and the operation is addition. See where we're heading with this? If we add an even number to another even number guess what we get - an even number!

Like:

$$2 + 2 = 4$$

I bet you already knew that!

## *2. Identity Element*

If you combine any member of the set with the identity element, you get back the original number. How great is that! I love things that don't change!

For example, let's say the set consists of some numbers and the operation is multiplication. If the identity element is the number 1, we can multiply any of these numbers by 1 and the result is always the number we started with.

$$a \times 1 = a$$

OK, this is not rocket science!

## *3. Inverse*

If you combine any member of the set with its inverse you get the identity element.

For example, let's say the set consists of all rational numbers. If we took the number 4 and multiply it by its inverse (1/4th), we would end up with the identity element, in this case the number 1. How crazy is that!

4 x 1/4 = 1

Hang in there for one more, and then we'll get back to hiking.

## *4. Associative*

Let's take three members of the set (call them a, b, and c). If the set is associative under this operation, it doesn't matter how we "associate" them.

We can apply the operation to (a and b), and then apply it to c, or we can apply it to (b and c) and then to a. No matter how you associate them, you will get the same result!

Let's say we are adding the integers 1, 2, and 3. We will get the same result if we add:

1. (1+2) + 3 = 3 + 3 = 6 or
2. 1 + (2+3) = 1 + 5 = 6

In more general terms:

$$(a * b) * c = a * (b * c)$$

So, it doesn't matter how we associate them. Again, not rocket science!

For those of you who like symbols, check this out:

$$\forall a, b, c \in G \exists * \ni (a * b) * c = a * (b * c)$$

How cool is that!

What this statement says is, "For all a, b and c contained in the set G there exists an operator called * such that (a * b) * c equals (or gives the same result as) a * (b * c)." I love it!

OK, let's go back out to the woods.

## *Let's Look at Our Hiking Group*

### The Elements and the Operation

While out hiking down the trail. You may:

1. Look to the right (R)
2. Look to the left (L)
3. Look behind you (B) or
4. Just enjoy the view looking straight ahead (I)

And the operation is "Followed by."

* = "Followed by"

Notice that the elements of this group are not numbers. Therefore, it is called a "non-numeric" group.

## *Closure*

Here is a table of all possible binary combinations. Notice that all the results end up back in the set. Therefore we have **Closure!**

| 1st Element | 2nd Element | 1 * 2 |
|---|---|---|
| R | R | B |
| R | L | I |
| R | B | L |
| R | I | R |
| L | R | I |
| L | L | B |
| L | B | R |
| L | I | L |
| B | R | L |
| B | L | R |
| B | B | I |
| B | I | B |
| I | R | R |
| I | L | L |
| I | B | B |
| I | I | I |

**Figure 1: For All Possibilities We End up Back in the Set**

## 2. Identity Element

The Identity Element is to do nothing, or just look straight ahead.

## 3. Inverse

Each movement has an inverse. Here is a table summarizing the inverses:

| Element | Inverse | Result |
|---|---|---|
| R | L | I |
| L | R | I |
| B | B | I |
| I | I | I |

## 4. Associative

Here again, it doesn't matter how we associate these actions, the result will be the same. For example:

R * (R * L) = (R * R) * L = R
or
L * (B * R) = (L * B) * R = B

Happy Hiking!
Rick